Two evolutionary paths of an axisymmetric gravitational instability in the dust layer of a protoplanetary disk

Fumiharu Yamoto
National Astronomical Observatory of Japan

Minoru Sekiya
Kyushu University

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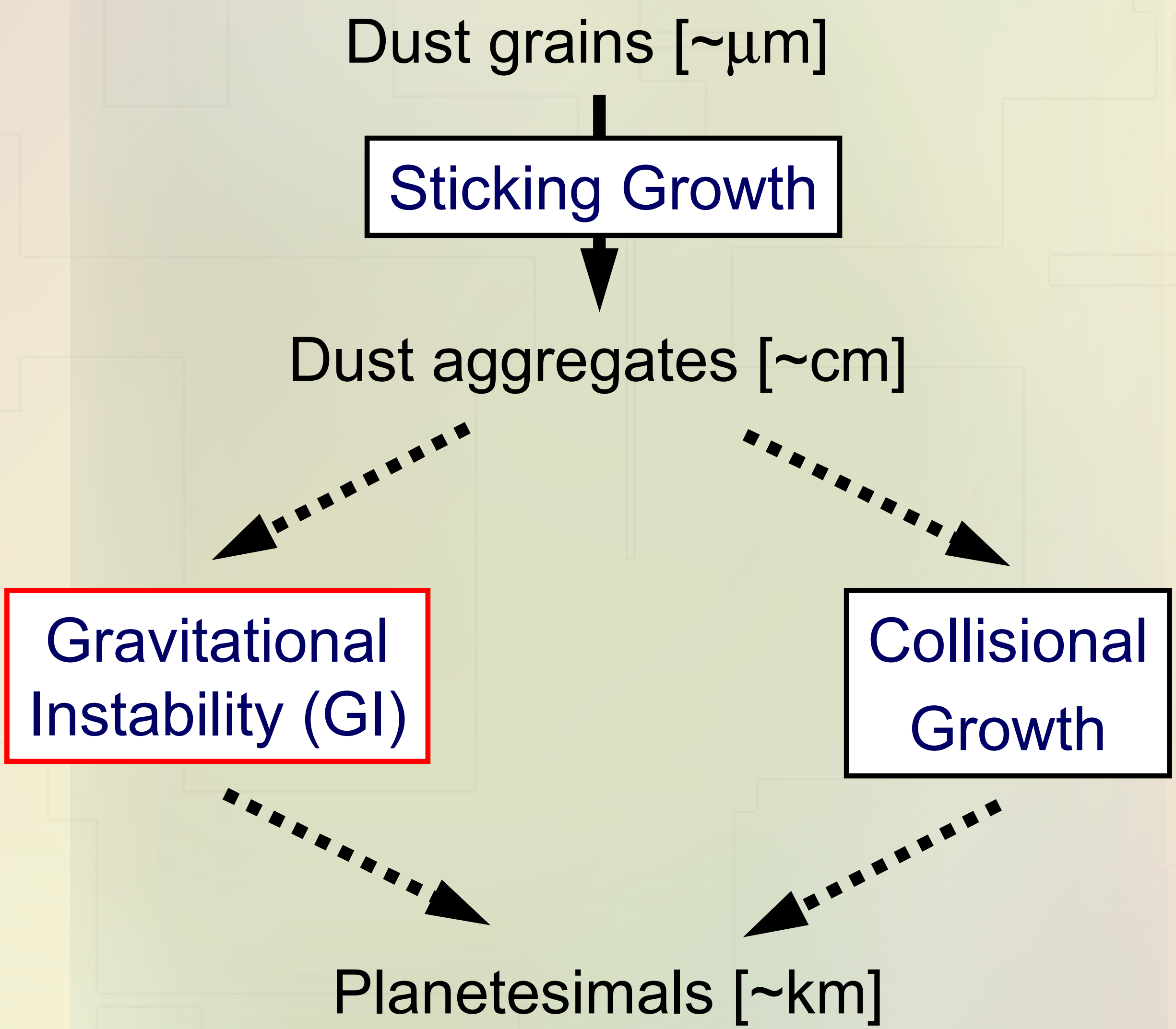
Introduction --- How are planetesimals formed?



Introduction --- Gravitational instability in a dust layer

Dust settling toward the midplane

↓

Critical state of GI

↓

GI growth

↓

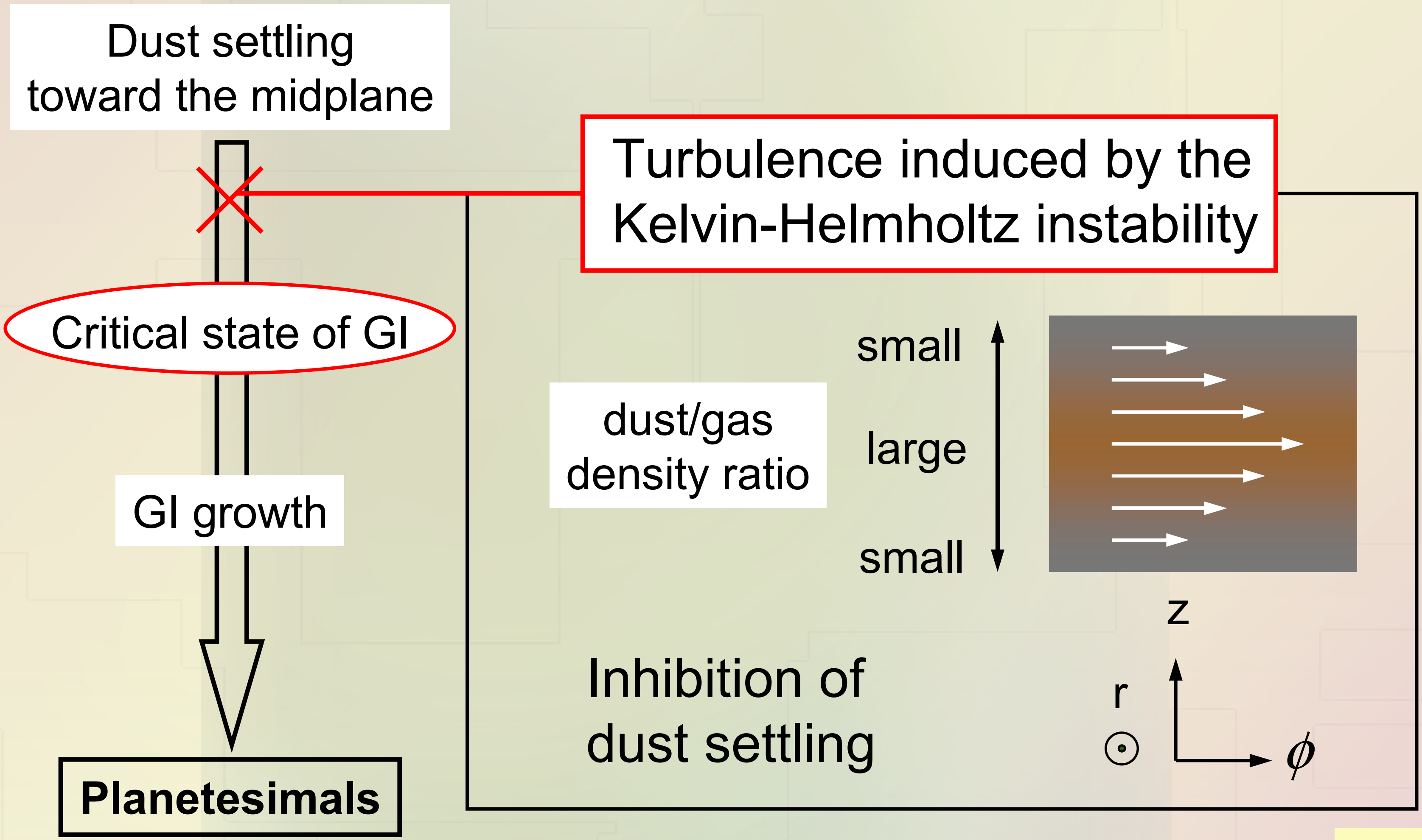
Planetesimals

Linear Analysis

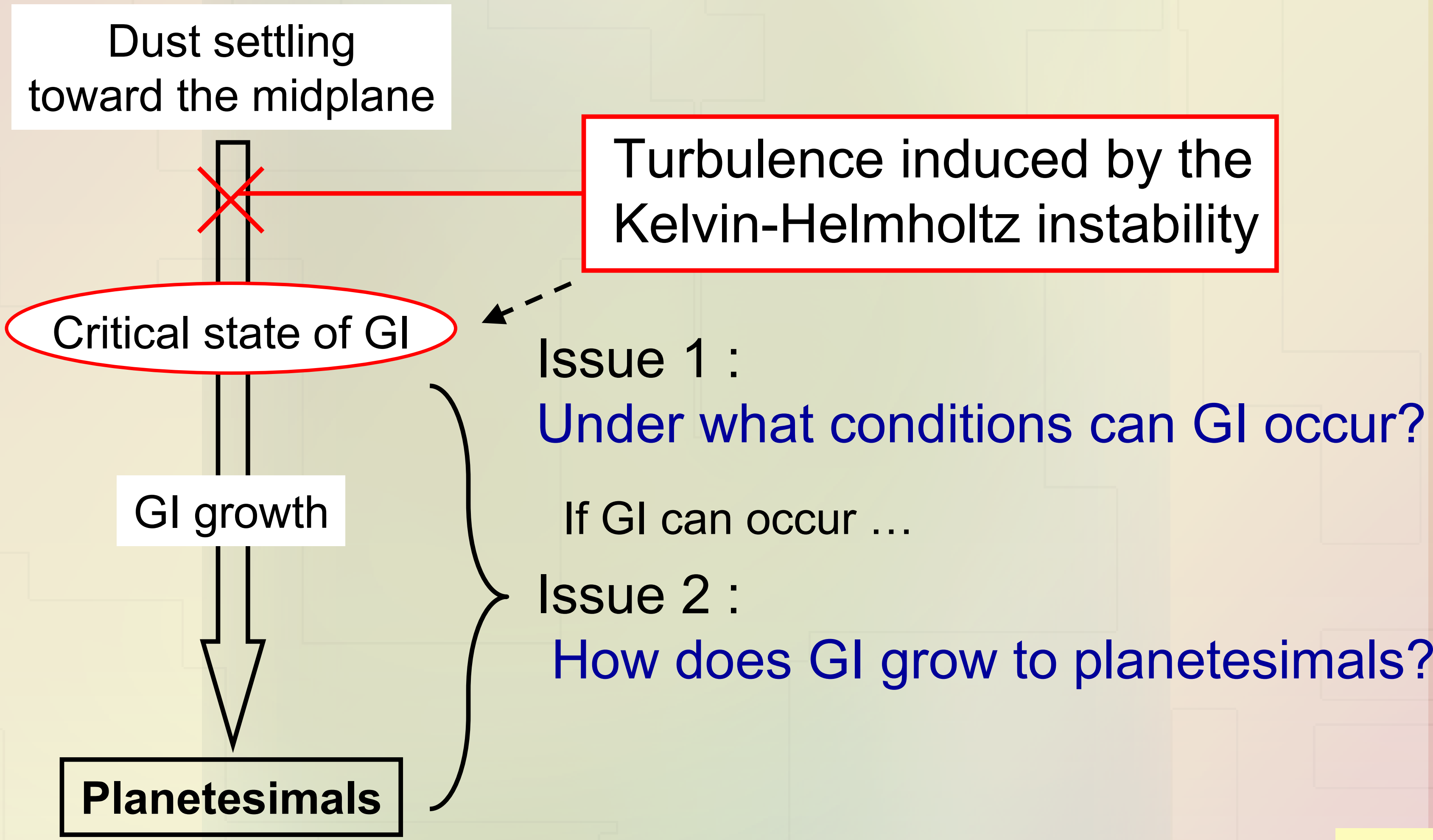
- Goldreich and Ward 1973
- Sekiya 1983
- Yamoto and Sekiya 2004

→ Size estimation of a planetesimal 1-10 km

Introduction --- Gravitational instability in a dust layer



Introduction --- Gravitational instability in a dust layer

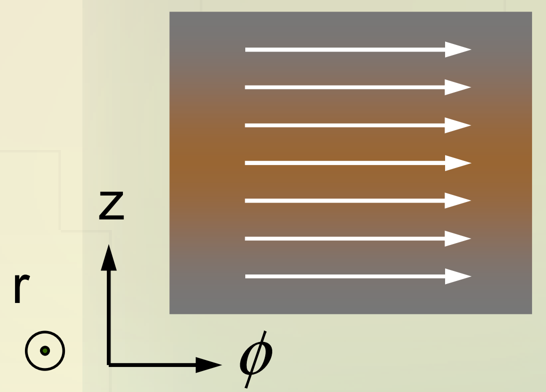


Introduction --- Gravitational instability in a dust layer

Three conditions to cause GI

1. Dust growth to ~10 m (Weidenschilling 1980)
   - Inhibition by a dispersion of radial velocities (Weidenschilling 1995)
2. Supersolar dust/gas surface density ratios (Sekiya 1998)
3. Region where the radial pressure gradient at equilibrium is negligible

The dust and gas rotates at Keplerian velocity independent of a dust/gas density ratio.



The Kelvin-Helmholtz instability doesn’t cause.

⇩

Dust aggregates continue to settle.
The critical state of GI is realized.

Introduction --- Gravitational instability in a dust layer

Three conditions to cause GI

1. Dust growth to ~10 m (Weidenschilling 1980)
   - Inhibition by a dispersion of radial velocities (Weidenschilling 1995)
2. Supersolar dust/gas surface density ratios (Sekiya 1998)
3. Region where the radial pressure gradient at equilibrium is negligible

We adopt the third condition and perform hydrodynamic numerical simulations of how does GI grow to several times the critical density.

Our Study --- Model assumptions and settings

- Assumptions
  - No global turbulence
  - Axisymmetric dust layer
  - A constant gas friction time: $\Omega_K t_f = 0.01$ or $\Omega_K t_f = 0.1$

    Ω_K : Keplerian angular velocity, t_f : gas friction time
  - The Gaussian dust density distribution

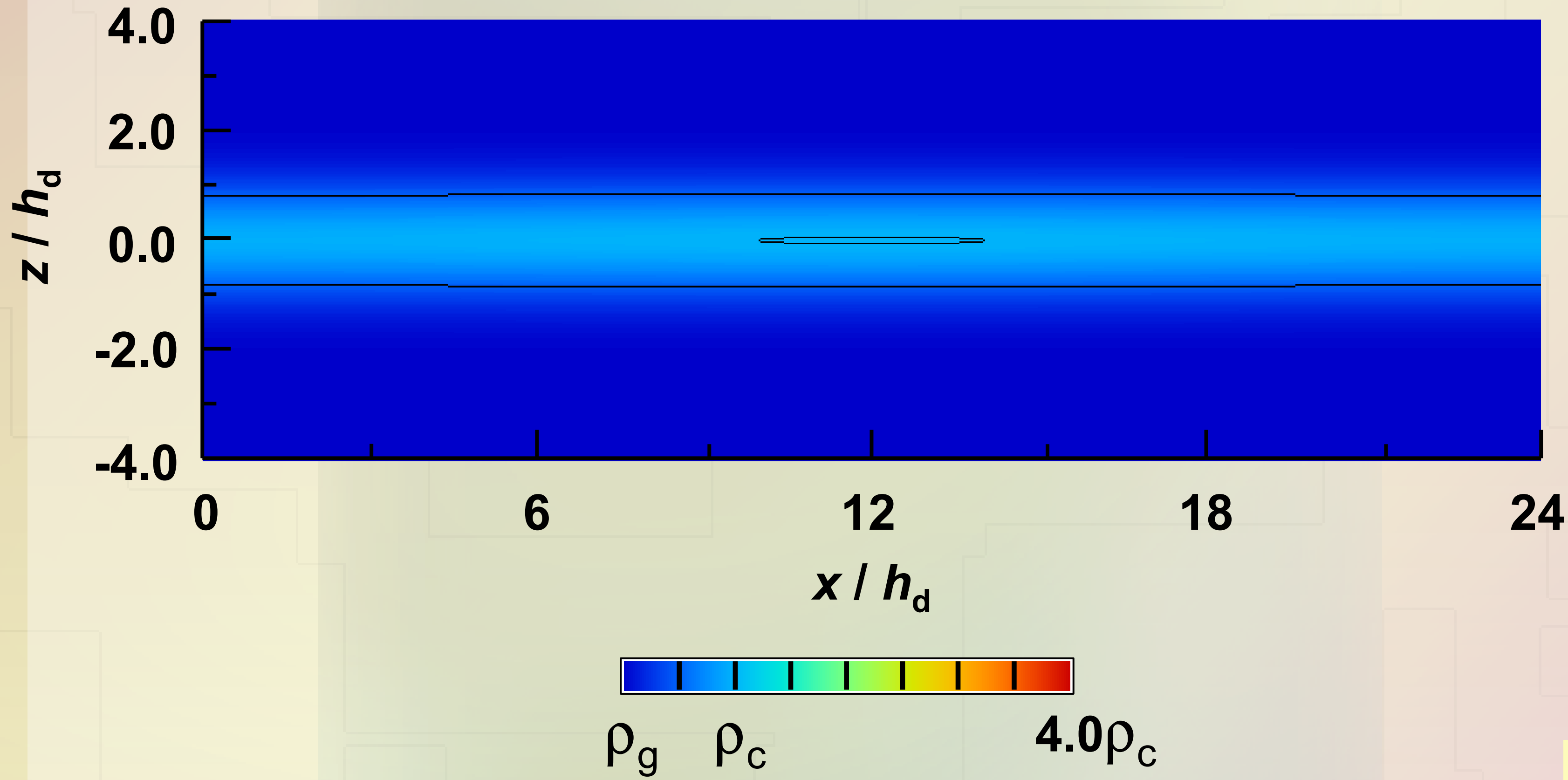
- Settings
  - Two fluids consisting of dust fluid and gas
  - Local Cartesian coordinate system

Our Study --- Results of numerical simulations

Case 1 : $\Omega_K t_f = 0.01$

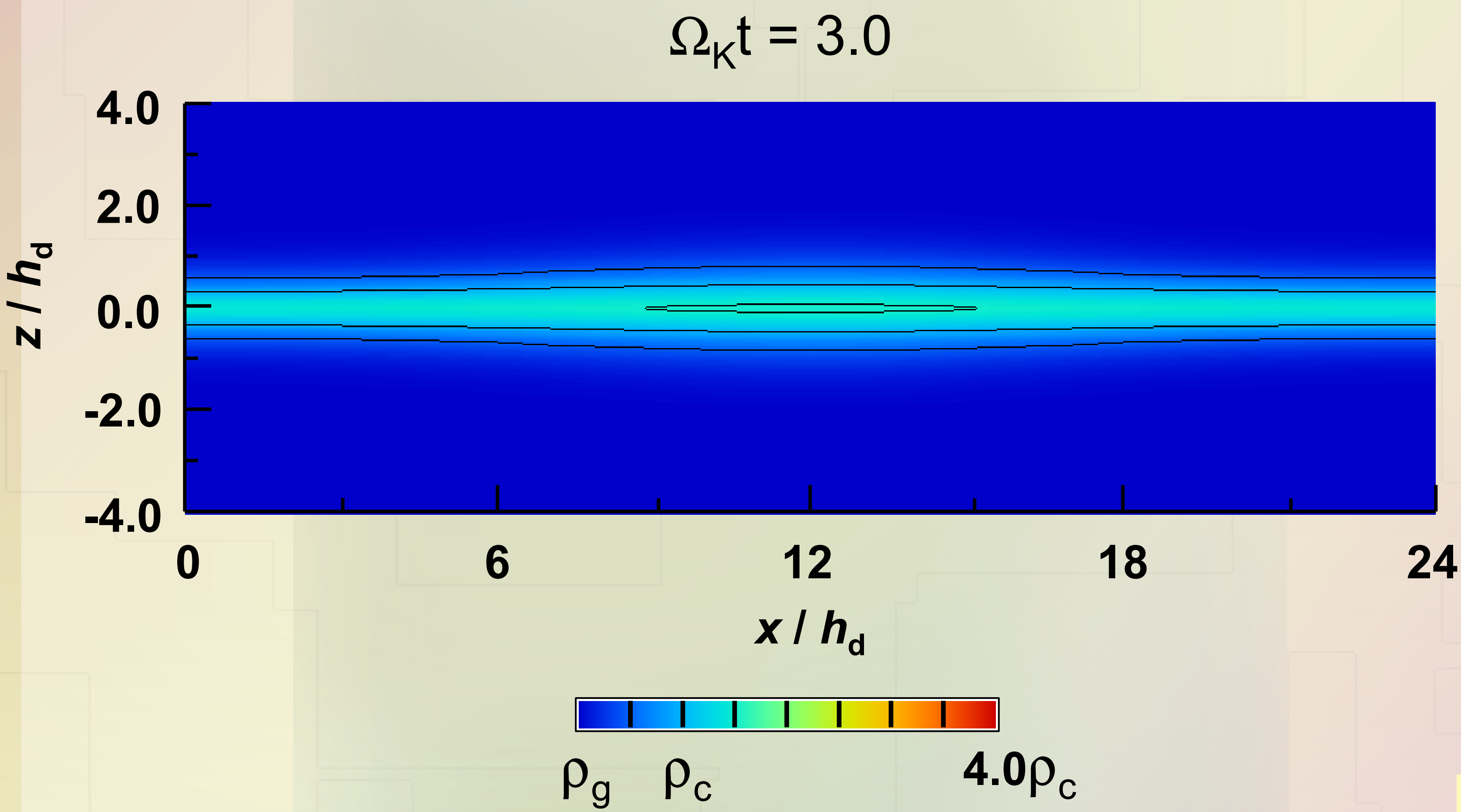
(ρ_g : gas density
ρ_c : critical density of GI
h_d : scale height of the dust layer)

$\Omega_K t = 0.0$



Our Study --- Results of numerical simulations

Case 1 : $\Omega_K t_f = 0.01$



Our Study --- Results of numerical simulations

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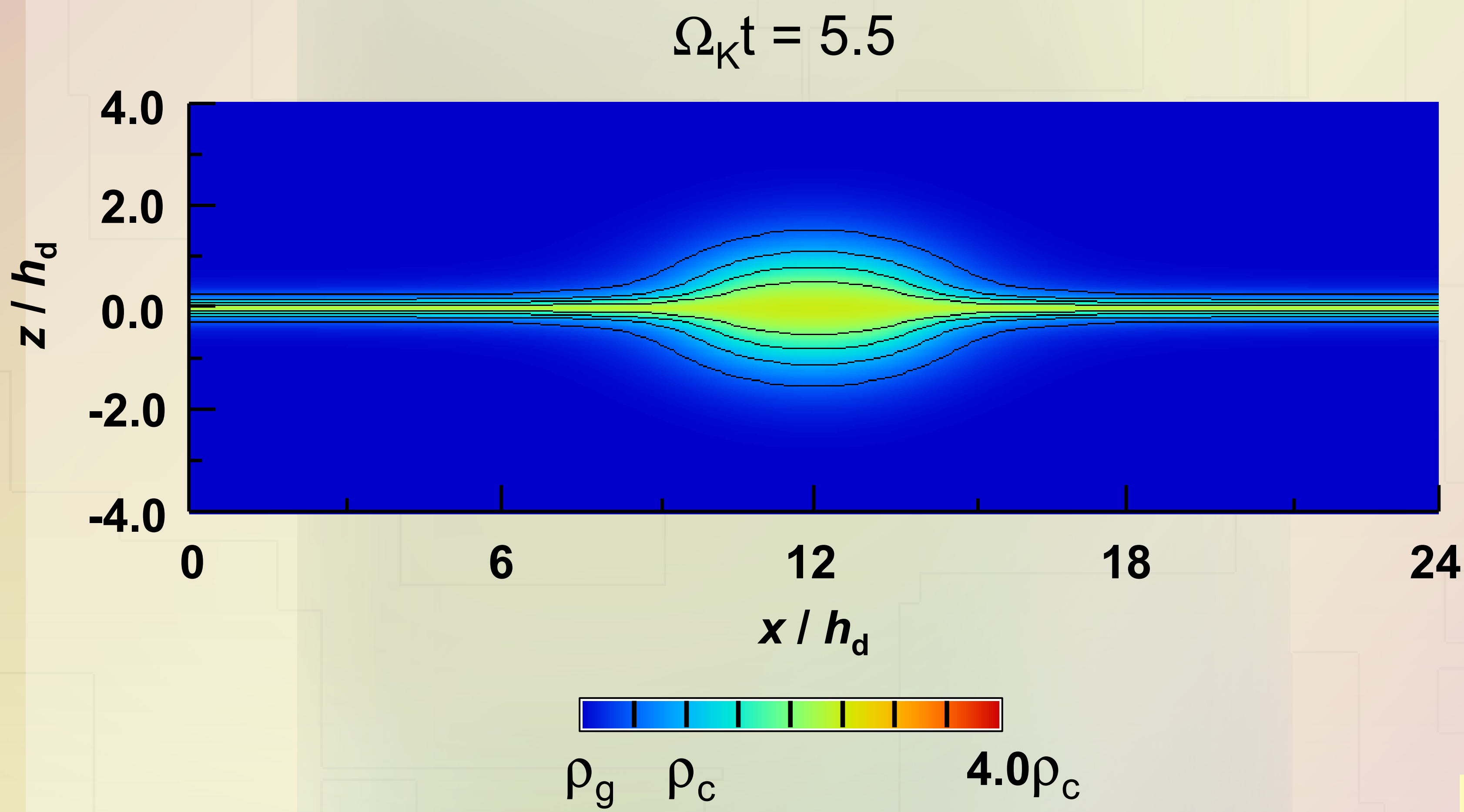
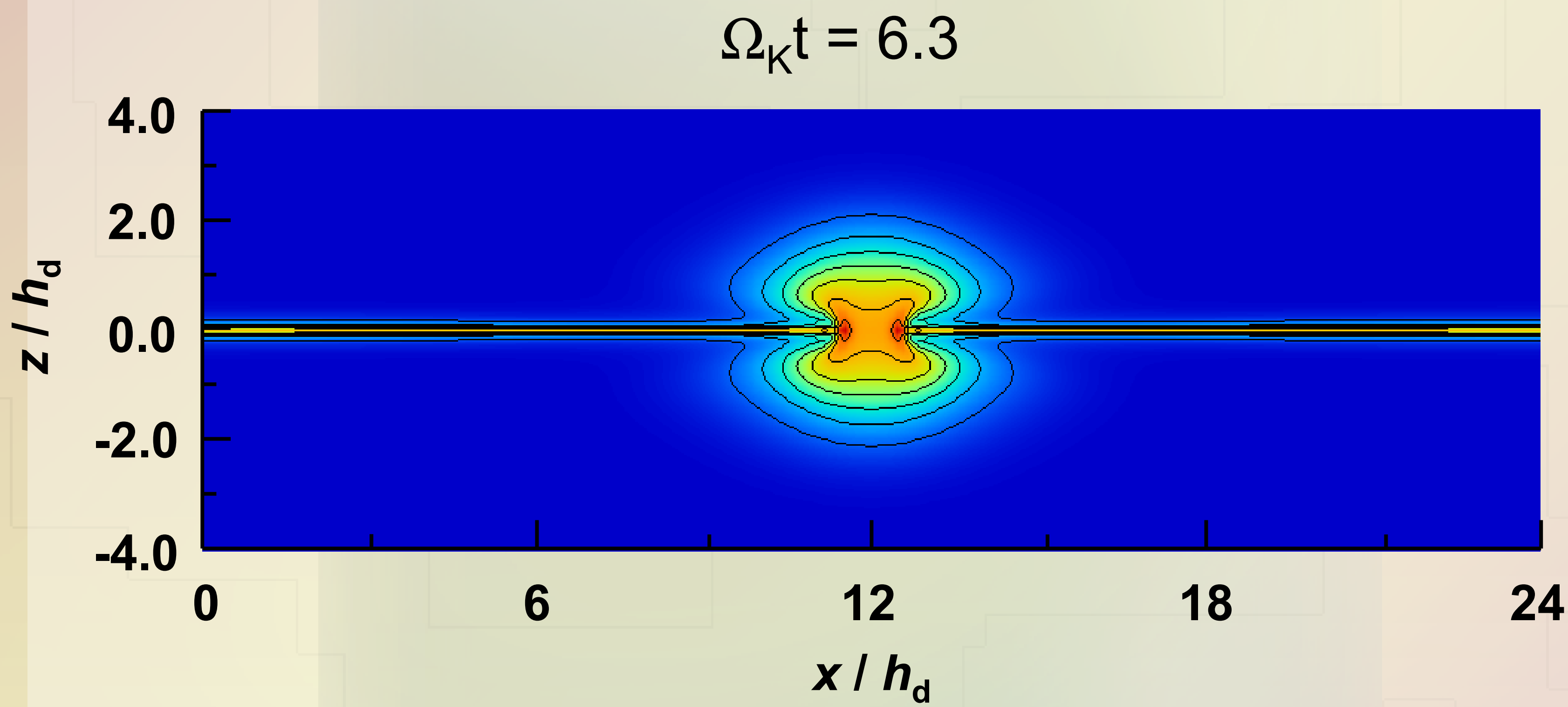


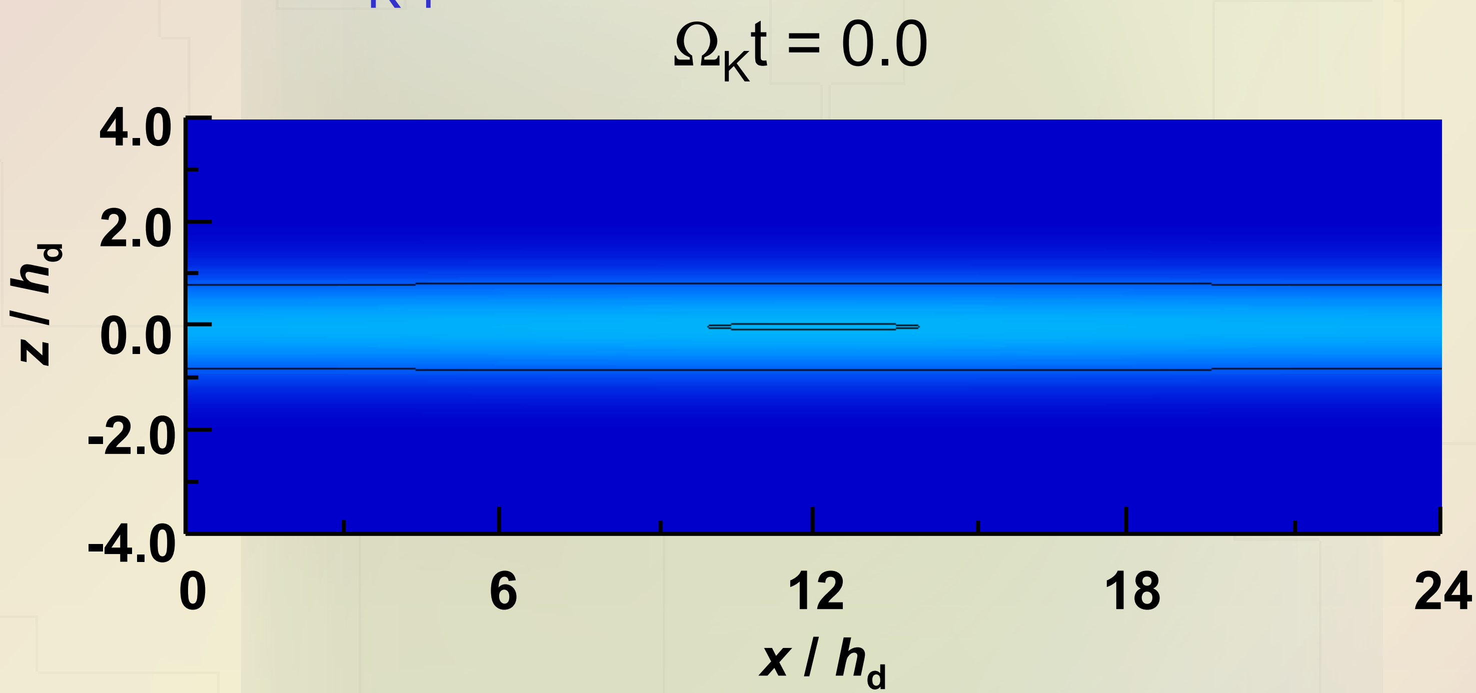
Our Study --- Results of numerical simulations

Case 1 : $\Omega_K t_f = 0.01$



Our Study --- Results of numerical simulations

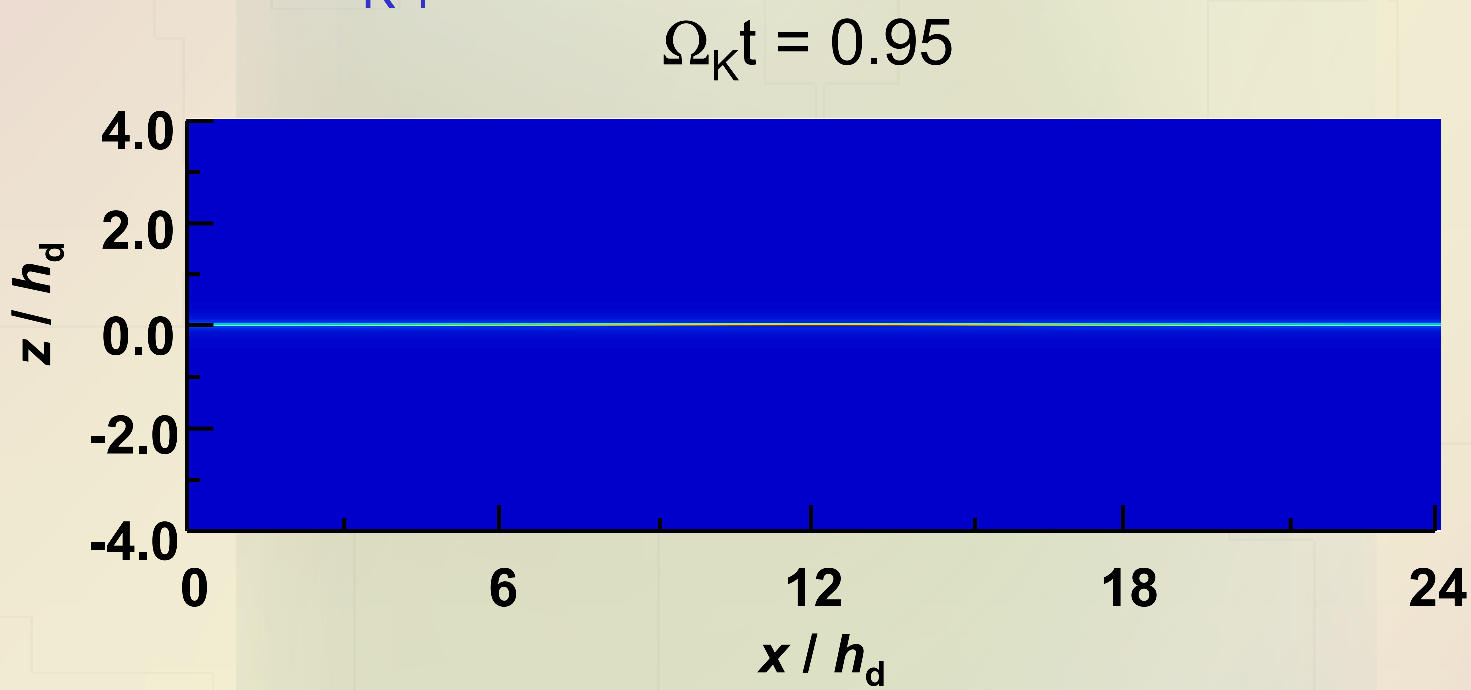
Case 2 : $\Omega_K t_f = 0.1$



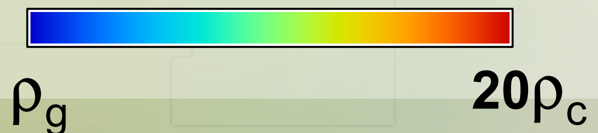
Density distribution : the same as the case of 0.01
Dust settling velocity : <u>ten times</u> the case of 0.01

Our Study --- Results of numerical simulations

Case 2 : $\Omega_K t_f = 0.1$

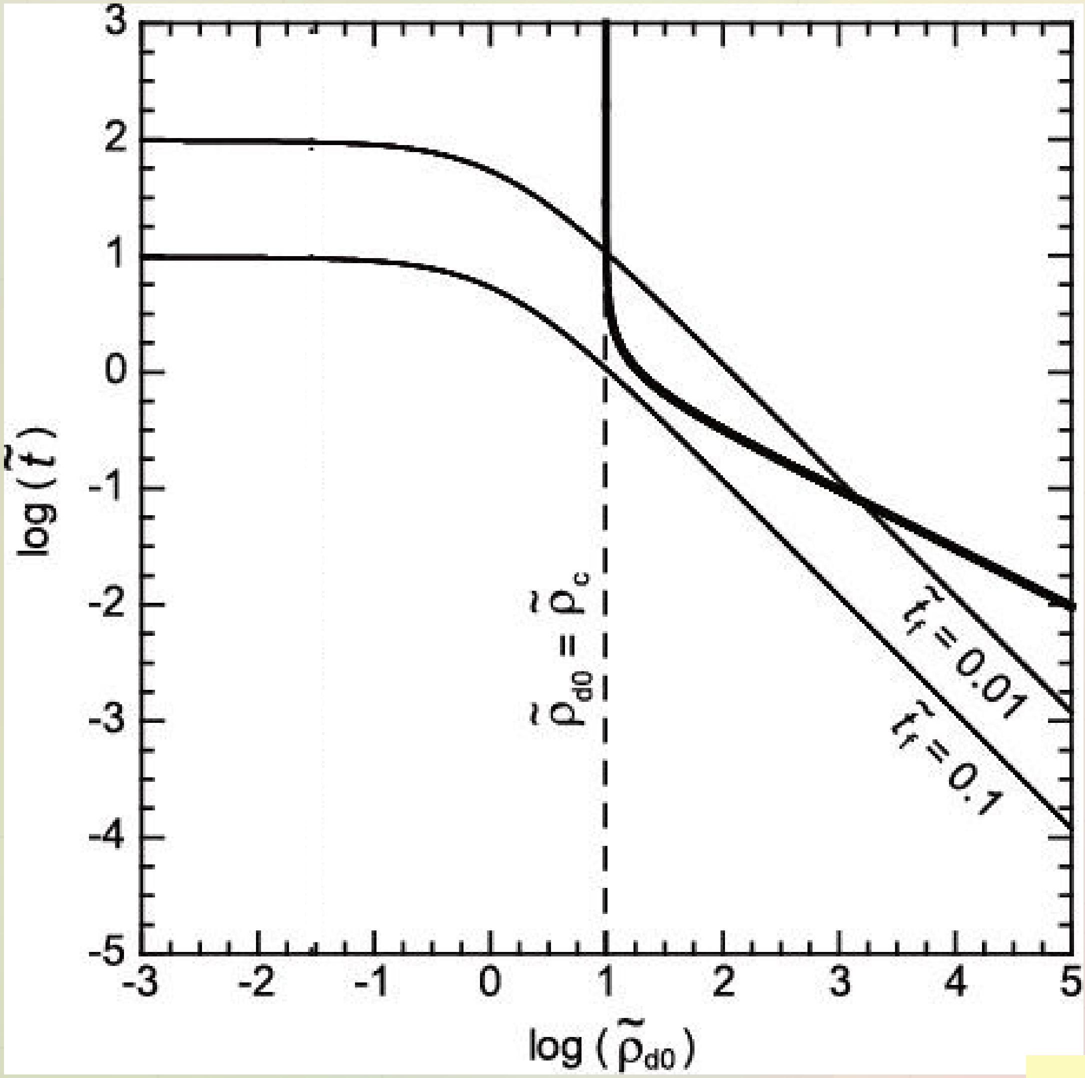


GI does not seem to grow, at least until twenty times the critical density.



Our Study --- Results of an analytical calculation

Dust settling time
(thin lines)
vs.
Growth time of GI
(thick line)



If $\Omega_K t_f = 0.1$,

Dust settling time is shorter than growth time of GI in all dust densities

Tilde denotes nondimensional values

Summary --- see also Yamoto & Sekiya, ApJ, 646, L155

$\Omega_K t_f = 0.01$

Gravitational instability grows faster than dust settling.

$\Omega_K t_f = 0.1$

Dust aggregates settle before gravitational instability grows, regardless of the dust density, and the dust layer will become extremely thin.

Future works

- Nonaxisymmetric (3D) numerical simulations
- Simulations under the conditions:
  - Different sizes of dust aggregates
  - Supersolar dust/gas surface density ratios

Thank you for your attention.

Gas friction time

$$t_f = \frac{m\,V}{F_d}$$

m : dust mass
V : velocity relative to the gas
F_d : drag force

For spherical dust particles

$\Omega_K t_f = 0.01$ ⟶ **a = 4 cm**, $\rho_s = 1\ g/cm^3$ at 1 AU
(Epstein drag regime)

$\Omega_K t_f = 0.1$ ⟶ **a = 13 cm**, $\rho_s = 1\ g/cm^3$ at 1 AU
(Stokes drag regime)

Gas

Incompressibility condition

$$\nabla \cdot \mathbf{v}_{\mathrm{g}} = 0$$

$$\mathbf{\Omega} = (0, 0, \Omega_{\mathrm{K}})$$

Equation of motion

$$\frac{\partial \mathbf{v}_{\mathrm{g}}}{\partial t} + (\mathbf{v}_{\mathrm{g}} \cdot \nabla)\mathbf{v}_{\mathrm{g}} = -\frac{1}{\rho_{\mathrm{g}}}\nabla P_{\mathrm{g}} + \begin{pmatrix} 3\Omega_{\mathrm{K}}^2 x \\ 0 \\ -\Omega_{\mathrm{K}}^2 z \end{pmatrix} - 2\mathbf{\Omega} \times \mathbf{v}_{\mathrm{g}} - \nabla \psi + \frac{\rho_{\mathrm{d}}}{\rho_{\mathrm{g}}} \frac{\mathbf{v}_{\mathrm{d}} - \mathbf{v}_{\mathrm{g}}}{t_{\mathrm{f}}}$$

Tidal force

Coriolis force

Self-gravity force

Gas pressure gradient force

Central star's gravity force

Back-reaction of gas drag force

Dust

Equation of continuity

$$\frac{\partial \rho_{\mathrm{d}}}{\partial t} + \nabla \cdot (\rho_{\mathrm{d}} \mathbf{v}_{\mathrm{d}}) = 0$$

Equation of motion

$$\frac{\partial \mathbf{v}_{\mathrm{d}}}{\partial t} + (\mathbf{v}_{\mathrm{d}} \cdot \nabla)\mathbf{v}_{\mathrm{d}} = \begin{pmatrix} 3\Omega_{\mathrm{K}}^2 x \\ 0 \\ -\Omega_{\mathrm{K}}^2 z \end{pmatrix} - 2\mathbf{\Omega} \times \mathbf{v}_{\mathrm{d}} - \nabla \psi + \frac{\mathbf{v}_{\mathrm{g}} - \mathbf{v}_{\mathrm{d}}}{t_{\mathrm{f}}}$$

Gas drag force

Poisson equation of self-gravitational potential

$$\nabla^2 \psi = 4\pi G(\rho_{\mathrm{g}} + \rho_{\mathrm{d}})$$