Two Evolutionary Paths of an Axisymmetric Gravitational Instability in the Dust Layer of a Protoplanetary Disk

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1. Introduction

There are mainly two models of planetesimal formation: one is the growth through mutual sticking of dust aggregates due to nongravitational forces, and another is the fragmentation of a dust layer due to gravitational instability (GI). In this poster, the latter model is considered. Nonlinear hydrodynamic numerical simulations are performed to investigate how GI grows to several times the critical density. The results of the simulations agree with those of our approximate analytical caluculation, and show two evolutionary paths of an axisymmetric GI, depending on gas friction time.

2. Disk Model and Assumptions

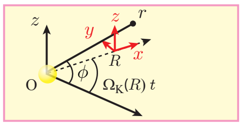
2-1. Two-fluid model of dust fluids and gas

All dust aggregates are assumed to have a constant gas friction time which is smaller than the Keplerian period, and the effect of randam velocities of dust aggregates is omitted. A collection of dust aggregates is considered as a dust fluid. Thus, two fluids consisting of dust fluids and gas are considered. The gas is assumed to be incompressible, since the scale height of the dust layer in this study is much less than that of the gas disk.

2-2. Local Cartesian coordinate system

A dust layer is assumed to be axisymmetric with respect to the rotation axis of a protoplanetary disk.

Instead of the inertial cylindrical coordinate system (r, ϕ, z), we use the local Cartesian coordinate system (x, y, z) around a fixed representative distance R from a central star rotating with the local Keplerian angular velocity $\Omega_K(R)$:



$$x = r - R,$$
$$y = R\,[\,\phi - \Omega_K t\,],$$
$$z = z,$$

where t is the time. Hereafter, $\Omega_K(R)$ is denoted by Ω_K for simplicity. The local approximation is used; i.e. higher order terms of (x, y, z) are neglected.

2-3. No radial pressure gradient at equilibrium

We assume the region where a radial pressure gradient at equilibrium is negligible, i.e. a pressure is maximum with respect to the radial distance from a central star is supposed (Haghighipour and Boss, 2003). Under this condition, shear-induced instability doesn't cause, and the critical state of GI is realized.

3. Formulation

3-1. Basic equations

$$\mathbf{\Omega} = (\,0\,,\,0\,,\,\Omega_K\,)$$

Gas component

Incompressibility condition :
$$\nabla\cdot\mathbf{v}_g = 0 \qquad (1)$$

Equation of motion (Euler's equation) :
$$\frac{\partial \mathbf{v}_g}{\partial t} + \mathbf{v}_g\cdot\nabla\mathbf{v}_g = -\frac{1}{\rho_g}\nabla P_g + 3\Omega_K^2 x\boldsymbol{i} - \Omega_K^2 z\boldsymbol{k} - 2\mathbf{\Omega}\times\mathbf{v}_g - \nabla\psi + \frac{\rho_d}{\rho_g}\frac{\mathbf{v}_d - \mathbf{v}_g}{t_f} \qquad (2)$$

Dust component

Equation of continuity :
$$\frac{\partial \rho_d}{\partial t} + \nabla\cdot(\rho_d \mathbf{v}_d) = 0 \qquad (3)$$

Equation of motion (Euler's equation)
$$\frac{\partial \mathbf{v}_d}{\partial t} + \mathbf{v}_d\cdot\nabla\mathbf{v}_d = 3\Omega_K^2 x\boldsymbol{i} - \Omega_K^2 z\boldsymbol{k} - 2\mathbf{\Omega}\times\mathbf{v}_d - \nabla\psi + \frac{\mathbf{v}_g - \mathbf{v}_d}{t_f} \qquad (4)$$

Poisson equation of the self-gravitational potential
$$\nabla^2\psi = 4\pi G\rho \qquad (5)$$

Symbols

- $\boldsymbol{i}$: Unit vector in the x direction
- $\mathbf{v}_g$: Gas velocity vector (u_g , v_g , w_g)
- ρ : Sum of the dust density ρ_d and the gas density ρ_g
- P_g : Nebular gas pressure
- t_f : Gas friction time
- $\boldsymbol{k}$: Unit vector in the z direction
- $\mathbf{v}_d$: Dust velocity vector (u_d , v_d , w_d)
- ψ : Self-gravitational potential

3-2. Initial condition

- Σ_d : Surface density of the dust
- ρ_c : Critical density of GI

3-2-1. Unperturbed state

Density distribution vertical to the midplane

Dust $\quad \rho_{d0}(z) = \rho_{d0} = \rho_c \exp(-z^2/h_d^2)$: The Gaussian distribution

$$h_d = \frac{\Sigma_d}{\sqrt{\pi}\rho_c}$$: Scale height of a dust layer

Gas $\quad \rho_{g0}(z) = \rho_{g0}(0) = \mathrm{const}$ for $|z| << h_g$ $\quad h_d/h_g \sim 1\times10^{-5}$

- h_g : Scale height of a gas layer

Velocity distribution

$$u_{g0} = 0\,,\ v_{g0} = -\frac{3}{2}\Omega_K x\,,\ w_{g0} = 0$$

$$u_{d0} = 0\,,\ v_{d0} = -\frac{3}{2}\Omega_K x\,,$$

Dust settling velocity : $w_{d0} = -\,t_f\left(\Omega_K^2 z + \frac{\partial\psi}{\partial z}\right)$

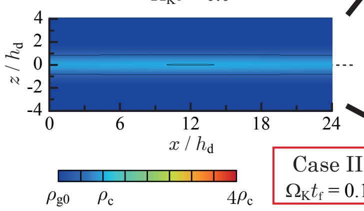
3-2-2. Perturbed quantities

$$\rho_{d1} = -\,\varepsilon\, Re[\rho_{d0}\exp(ik_c x)]\,,\quad \rho_{g1} = 0\,,\quad \mathbf{v}_{g1} = \mathbf{0}\,,\quad \mathbf{v}_{d1} = \mathbf{0}$$

4. Results

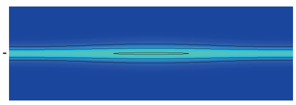
4-1. Numerical Simulations

($\varepsilon = 0.01$ in the perturbed dust density)

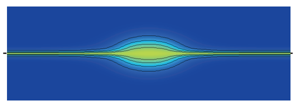


An initial density distribution of simulations. A total density of dust fluids and gas is shown as a color contour map. In the case II, velocities of dust settling is ten times values in the case I, because they are proportional to gas friction time.

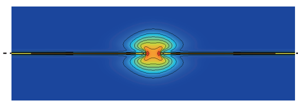
Case I $\quad \Omega_K t_f = 0.01$



Dust settling increases the density near the midplane, while the growth of GI is still not significant.

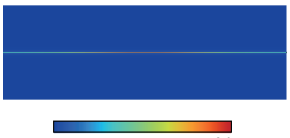


GI grows faster than dust settling. The dust layer changes into a ring circling a central star.



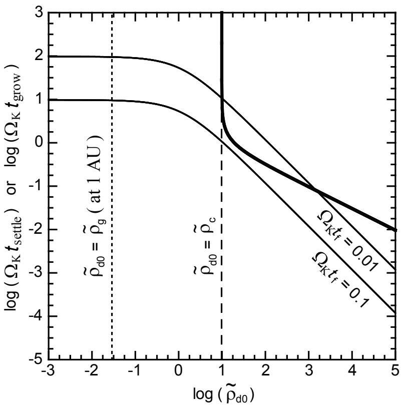
The density have a maximum value on the left and right in the ring. Concave density distributions arise near the density maxima.

Case II $\quad \Omega_K t_f = 0.1$



Dust settling is faster than the growth of GI, at least until twenty times the critical denisty. Note that only this figure has a different color contour map.

4-2. Analytical Calculation



Nondimensional density : $\tilde{\rho} = 4\pi G\rho\,/\,\Omega_K^2$

This figure compares the dust settling time (thin lines) with the growth time of GI (thick line). The dotted and dashed lines show the values that the dust density at the midplane is equal to the gas density at 1 AU and the critical density, respectively.

If the dust density exceeds the gas density, the dust settling time shortens by the self-gravity of the dust layer. If $\Omega_K t_f = 0.01$, the growth time becomes shorter than the dust settling time after the density exceeded the critical density. If $\Omega_K t_f = 0.1$, the dust settling time is shorter than the growth time for all dust densities.

The results agree with those of numerical simulations.

5. Summary

If $\Omega_K t_f = 0.01$, GI grows faster than dust settling. On the other hand, if $\Omega_K t_f = 0.1$, dust settling is faster than GI for all dust densities. Thus, the dust layer will become extremely thin. GI will grow with very short wavelengths after dust settling almost ceased, and then modes with larger wavelengths may begin to grow slowly, as suggested by Goldreich and Ward (1973). Therefore, two evolutionary paths is found under the condition in this study.