Local Axisymmetric Two-Fluid Simulations of the Gravitational Instability in the Dust Layer of a Protoplanetary Disk

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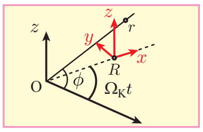
1. Introduction

The fragmentation of the dust layer due to the gravitational instability (GI) is one of possible models of the planetesimal formation. We previously obtained the critical density ρ_c of the GI in the dust layer of a protoplanetary disk with nonuniform dust density distributions in the direction vertical to the midplane using the linearized equations (Yamoto and Sekiya, 2004, Icarus, *170*, 180-192). Here, we report the nonlinear evolution of axisymmetric modes after the density in the dust layer exceeds the critical value of the GI. We perform numerical simulations using the two-fluid model, in which the dust and gas are considered separately.

2. Assumptions and Disk Model

2-1. Local Cartesian coordinate system

Instead of the inertial cylindrical coordinate system (r, ϕ, z), we use the local Cartesian coordinate system (x, y, z) around a fixed representative distance R from the central star rotating with the local Keplerian angular velocity $\Omega_K(R)$:



$$x = r - R\,,\quad y = R[\phi - \Omega_K(R)t]\,,\quad z = z\,,$$

where t is the time. Hereafter, $\Omega_K(R)$ is denoted by Ω_K for simplicity. The local approximation is used; i.e. higher order terms of (x, y, z) are neglected.

The dust layer is assumed to be axisymmetric with respect to the rotation axis of the disk. Thus all the physical quantities are independent of y.

2-2. Radial pressure gradient of the unperturbed state

We assume that the nebula is laminar. The radial pressure gradient of the unperturbed state is assumed to be zero, i.e. the region where the pressure is maximum with respect to the radial distance from the central star is supposed (Haghighipour and Boss, 2003), because the turbulence induced by the shear instability prevents the dust settling and the dust density never reaches the critical value of the GI for the typical value of the radial pressure gradient for the minimum mass disk.

2-3. Gas friction time

All dust aggregates have the same gas friction time which is constant for time.

We assume that the gas friction time is shorter than the Keplerian period and the random motion of dust aggregates (dust pressure) is negligible.

2-4. Incompressibility of the gas

The gas is assumed to be incompressible since the scale height of the dust layer which is gravitationally unstable h_d is much less than that of the gas disk h_g.

3. Formulation and the Numerical Method

$$\boldsymbol{\Omega} = (0, 0, \Omega_K)$$

3-1. Basic equations

Gas component

The incompressibility condition :

$$\nabla \cdot \mathbf{v}_g = 0 \qquad (1)$$

The equation of motion (Euler's equation) :

$$\frac{\partial \mathbf{v}_g}{\partial t} + \mathbf{v}_g \cdot \nabla \mathbf{v}_g = -\frac{1}{\rho_g}\nabla P_g + 3\Omega_K^2 x\boldsymbol{i} - \Omega_K^2 z\boldsymbol{k} - 2\boldsymbol{\Omega}\times\mathbf{v}_g - \nabla\psi + \frac{\rho_d}{\rho_g}\frac{\mathbf{v}_d - \mathbf{v}_g}{t_f} \qquad (2)$$

Dust component

The equation of continuity :

$$\frac{\partial \rho_d}{\partial t} + \nabla \cdot (\rho_d \mathbf{v}_d) = 0 \qquad (3)$$

The equation of motion (Euler's equation)

$$\frac{\partial \mathbf{v}_d}{\partial t} + \mathbf{v}_d \cdot \nabla \mathbf{v}_d = 3\Omega_K^2 x\boldsymbol{i} - \Omega_K^2 z\boldsymbol{k} - 2\boldsymbol{\Omega}\times\mathbf{v}_d - \nabla\psi + \frac{\mathbf{v}_g - \mathbf{v}_d}{t_f} \qquad (4)$$

The Poisson equation of the self-gravitational potential

$$\nabla^2\psi = 4\pi G\rho \qquad (5)$$

Symbols

		t_f	the gas friction time
$\boldsymbol{i}$	the unit vector in the x direction	$\boldsymbol{k}$	the unit vector in the z direction
$\mathbf{v}_g$	the gas velocity vector	$\mathbf{v}_d$	the dust velocity vector
ρ	the sum of the dust density ρ_d and the gas density ρ_g		
P_g	the nebular gas pressure	ψ	the self-gravitational potential

3-2. Initial condition

Σ_d : the surface density of the dust

3-2-1. Unperturbed state

The density distribution vertical to the midplane $\rho_0(z) = \rho_{d0}(z) + \rho_{g0}(z)$

The dust $\rho_{d0}(z) = \rho_c \exp(-z^2/h_d^2)$: The Gaussian distribution

$h_d = \dfrac{\Sigma_d}{\sqrt{\pi}\rho_c}$: The scale height of the dust layer

The gas $\rho_{g0}(z) = \rho_{g0}(0) = \mathrm{const}$ for $|z| << h_g$ $\quad h_d/h_g \sim 1\times10^{-5}$

The velocity distribution

$\mathbf{v}_{g0} = \mathbf{0}\,,\quad u_{d0} = 0\,,\quad v_{d0} = 0\,,$

The dust settling velocity

$$w_{d0} = w_{g0} - t_f\left(\Omega_K^2 z + \frac{\partial\psi}{\partial z}\right)$$

3-2-2. Perturbed quantities

$$\rho_{d1} = -\,\varepsilon\, Re[\rho_{d0}\exp(ik_c x)]\,,\quad \rho_{g1} = 0,\quad \mathbf{v}_{g1} = \mathbf{v}_{d1} = \mathbf{0}$$

4. Results (Density evolution by the GI and the dust settling)

Parameters :

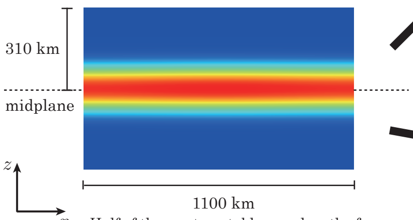
R = 1 AU
Σ_d = 7.1 g/cm²
ε = 0.01

Case I

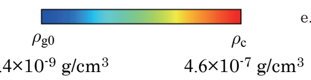
Gas friction time : $t_f = 2.6\times10^{-3}$ year

e.g. (dust aggregate radius : 3 cm, solid density : 3 g/cm³, porosity : 0 %)

t = 0 year

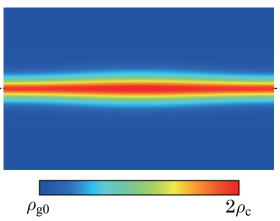


Half of the most unstable wavelength of the GI in our one-fluid linear solution (Yamoto and Sekiya, 2004)



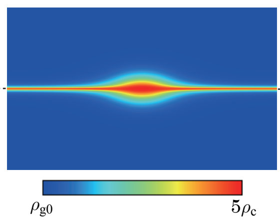
The initial density distribution of simulations. The color contour shows the total density of the dust and gas.

t = 0.4 year



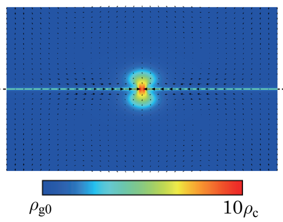
Dust aggregates settle toward the midplane and the dust density around midplane increases.

t = 0.7 year



The unstable mode of the GI grows.

t = 0.8 year



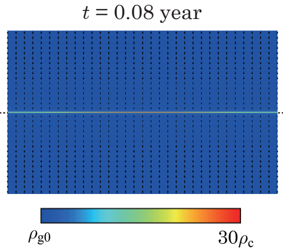
The concave density distributions arise right and left of the density maximum around the midplane. Arrows show the velocity vector.

Case II

Gas friction time : $t_f = 2.9\times10^{-2}$ year

e.g. (dust aggregate radius : 10 cm, solid density : 3 g/cm³, porosity : 0 %)

t = 0.08 year



Dust aggregates settle earlier than the growth of the GI. Arrows show the velocity vector.

5. Summary

In Case I, the density perturbation grows as dust aggregates settle; when dust aggregates concentrate to the center of x-axis on the midplane, concaves are seen right and left of the density maximum. On the other hand, the dust settling proceeds earlier than the growth of the GI in Case II. Therefore, the value of the gas friction time, in other words, of the dust aggregate radius when the dust layer becomes marginally unstable for the GI would be important for the evolution of the dust layer.

6. Future Work

We plan to perform nonaxisymmetric 3D simulations in order to elucidate the evolution after the growth of the axisymmetric mode.